Shear Alfvén vortices in a current-carrying low-temperature plasma

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Quasistationary structures associated with nonlinear shear Alfvén waves in low-β current-carrying plasmas are investigated. Dipole vortices with curved trajectories are found.

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I. INTRODUCTION

Recently, extensive studies on the nonlinear behavior of low-frequency waves in magnetized plasmas have been carried out [1–6]. It has been shown that at finite amplitudes these waves can self-organize into localized vortexlike structures through nonlinear interactions [6–10]. The structures, whose motion is perpendicular to the external magnetic field, have been invoked to explain the nearly stationary localized electromagnetic zones observed in the ionosphere and magnetosphere of the Earth [11]. Moreover, they could play an important role in the anomalous transport [7,12–14] near the edges of fusion plasmas, since they can trap the plasma particles in clusters and convectively move them across the magnetic field lines. It is of interest to point out that mathematically similar (they depend on the same vector nonlinearity) vortices can also appear in uncharged fluids as modons [12,13].

Most earlier studies [3–7] on electromagnetic vortex structures were carried out in the local, or plane, geometry and for current-free plasmas. Recently, Chen and Liu [15] investigated nonlinear shear Alfvén waves in a current-carrying homogeneous low-temperature plasma. They showed that the field-aligned external current can considerably affect the properties of the vortices. For low-β ($\beta \ll m_e/m_i \ll 1$) and low-current (with the steady-state electron flow speed much less than the Alfvén speed) plasmas, the existence domain for the propagation velocity of the vortices was found to be larger than that in the current-free case. In this paper, we extend the investigation of Ref. [15] to include nonlocal effects. In particular, we are interested in vortices having curved trajectories in a low-temperature current-carrying plasma. An electromagnetic vortex with both symmetric and antisymmetric parts is shown to exist.

This paper is organized as follows. In Sec. II, we derive the equations describing the nonlinear shear Alfvén waves in a low-β current-carrying plasma in the cylindrical geometry. In Sec. III, we obtain a vortex-like quasistationary solution describing an asymmetrical vortex revolving around the cylinder axis in a circular orbit. In the Conclusion, a discussion of the properties of this vortex and possible applications of the latter in space and fusion plasmas are given.

II. THE EVOLUTION EQUATIONS

We assume that the steady-state plasma is homogeneous and imbedded in a uniform magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$, where $\mathbf{e}_z$ is the unit vector in the z direction. The electrons are assumed to move along $\mathbf{B}_0$ at a constant speed $V_0\mathbf{e}_z$ relative to the ions.

For a low-β ($\beta \ll 1$) plasma, we can ignore the compressional part of the perturbed magnetic field. Thus, the perturbed electromagnetic fields of the shear Alfvén waves can be expressed as

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\partial_t A_z \mathbf{e}_z, \tag{1}$$

$$\mathbf{B}_\perp = \nabla A_z \times \mathbf{e}_z, \tag{2}$$

where ϕ and A_z are the scalar potential and the parallel component of the vector potential, respectively.

In the drift approximation ($\partial_t \ll \omega_{ci}$, where $\omega_{ci} = eB_0/cm_i$ is the ion gyrofrequency), the perpendicular (to the external magnetic field) fluid velocities of the ions and electrons are given by

$$\mathbf{V}_{e\perp} = \frac{c}{B_0}\mathbf{e}_z \times \nabla\phi + \frac{V_0\mathbf{B}_\perp}{B_0} + \frac{V_{ez}\mathbf{B}_\perp}{B_0} \tag{3}$$

and

$$\mathbf{V}_{i\perp} = \frac{c}{B_0}\mathbf{e}_z \times \nabla\phi - \frac{c}{B_0\omega_{ci}}\left[\partial_t + \frac{c}{B_0}(\mathbf{e}_z \times \nabla\phi)\cdot\nabla\right]\nabla\phi. \tag{4}$$

The first term on the right-hand side of (3) is the elec-

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tric drift, the second describes the coupling of the external current and the perturbed magnetic field, and the third comes from the nonlinear coupling of the parallel motion of the electrons and the perturbed magnetic field. The electron polarization drift has been neglected.

From the parallel component of the Ampere's law, neglecting the parallel motion of the ions as well as the displacement current, one obtains [15] a relation between the parallel electron velocity V_{ez} and the parallel vector potential A_z

$$V_{ez} = \frac{c}{4\pi e n_0}\Delta A_z, \quad (5)$$

where $\Delta_\perp = \partial^2/\partial r^2 + r^{-1}\partial/\partial r + r^{-2}\partial^2/\partial\theta^2$, and n_0 is the unperturbed density of the plasma.

We shall also need the charge conservation equation in the quasineutral limit and the parallel electron momentum equation [3,6,15], namely

$$\nabla \cdot \mathbf{J}_\perp + \partial_z J_z = 0, \quad (6)$$

and

$$\partial_t V_{ez} + (V_0 + V_{ez})\partial_z V_{ez} + \mathbf{V}_{e\perp} \cdot \nabla V_{ez} = -\frac{e}{m}E_z - \frac{e}{mc}(\mathbf{V}_{e\perp} \times \mathbf{B}_\perp)\cdot \mathbf{e}_z, \quad (7)$$

where we have assumed that the plasma is of low temperature such that $\beta \ll m_e/m_i$. The latter relation can be written as $\rho_s \ll \lambda_s$, where $\rho_s = c_s/\omega_{ci}$ (with c_s the ion acoustic speed) is the ion Larmor radius and $\lambda_s = c/\omega_{pe}$ is the collisionless electron skin depth. Thus, the inertial force dominates over the pressure force in the parallel electron momentum balance [3,4,6], and the problem under consideration does not explicitly involve the perturbed electron density.

Substituting (1)–(5) into (6) and (7), and noting that $\mathbf{V}_{e\perp}\cdot\nabla \gg V_{ez}\partial_z$, we can obtain the following coupled system of nonlinear evolution equations governing finite amplitude shear Alfvén waves:

$$[\partial_t + (\mathbf{e}_z \times \nabla\Phi)\cdot\nabla]\,\Delta\Phi + [\partial_z - (\mathbf{e}_z \times \nabla\Psi)\cdot\nabla]\,\Delta\Psi = 0 \quad (8)$$

and

$$\partial_t\Psi - [\partial_t + (\mathbf{e}_z \times \nabla\Phi)\cdot\nabla]\,\Delta\Psi + [\partial_z - (\mathbf{e}_z \times \nabla\Psi)\cdot\nabla]\,\Phi - V_0\,[\partial_z - (\mathbf{e}_z \times \nabla\Psi)\cdot\nabla]\,\Delta\Psi = 0. \quad (9)$$

In the above, we have normalized the variables as follows: $e\phi/T_e \rightarrow \Phi$, $eV_A A_z/cT_e \rightarrow \Psi$, $\omega_{ci}t \rightarrow t$, $cz/\omega_{pi} \rightarrow z$, $r/\lambda_s \rightarrow r$, $V_0/c \rightarrow V_0$, where $V_A = (B_0^2/4\pi n_0 m_i)^{1/2}$ is the Alfvén speed. In the small amplitude limit, we can obtain from (8) and (9) the linear dispersion relation $\omega = k_z V_A/(1 + k_\perp^2\lambda_s^2)^{1/2}$ for Alfvén waves in a low-β ($\beta \ll m_e/m_i$) plasma [3,4]. On the other hand, for $V_0 \rightarrow 0$, the above system reduces to that for a current-free plasma [6]. Note that the equilibrium electron flow causes not only a linear shift of the perturbation current, but also a nonlinear cross-field shift of the latter.

III. LOCALIZED VORTEX SOLUTIONS

In order to study quasistationary vortex structures in the cylindrical geometry, we assume that Φ and Ψ are functions of r and $\Theta = \theta - \Omega t + \alpha z$, where Ω is the angular speed at which the structure executes a circular orbit at a distance R from the axis, and α is a constant related to the parallel (to $\mathbf{B}_0$) characteristic length (e.g., $k_\parallel$) of the structure. That is, we are interested in a vortex structure which is infinite and that has a wavelength $2\pi/\alpha$ in the z direction. In this case, (8) and (9) can be written as

$$\left[\widetilde{\Phi},\ \Delta\Phi\right] - \left[\widetilde{\Psi},\ \Delta\Psi\right] = 0 \quad (10)$$

and

$$\left[\widetilde{\Phi},\ \Delta\Psi - \widetilde{\Psi}\right] - V_0\left[\widetilde{\Psi},\ \Delta\Psi\right] = 0, \quad (11)$$

where we have introduced for convenience the temporary notations $\widetilde{\Phi} \equiv \Phi - \frac{1}{2}\Omega r^2$ and $\widetilde{\Psi} \equiv \Psi - \frac{1}{2}\alpha r^2$. The Poisson's brackets $[f,\ g]$ are defined by

$$[f,\ g] \equiv \frac{1}{r}\left[\frac{\partial f}{\partial r}\frac{\partial g}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial g}{\partial r}\right].$$

According to the properties of the Poisson brackets, (10) and (11) are satisfied if

$$\Delta\Phi = f(\widetilde{\Phi}) - C_1\widetilde{\Psi} \quad (12)$$

and

$$\Delta\Psi = C_1\widetilde{\Phi} + (1 - C_1V_0)\widetilde{\Phi}, \quad (13)$$

where C_1 is a constant to be determined and $f(\widetilde{\Phi})$ is an arbitrary continuous function of $\widetilde{\Psi}$. For simplicity, we shall use a linear dependence, namely,

$$f(\widetilde{\Phi}) = C_0 + C_2\widetilde{\Psi},$$

where C_0 are C_2 are constants to be determined.

In order to investigate quasistationary structures having curved trajectories, we now transform the coordinate system (r, Θ) to a new one (r', θ') whose origin is at the center (located at $r = R$ from the axis of the original cylindrical coordinate system) of the structure [16,17]. Here, θ' is the angle between the tangent of the trajectory of the center of the structure and the radius vector of the point being considered. That is, $r^2 = r'^2 + 2Rr'\sin\theta' + R^2$. In contrast to the earlier studies on vortices in rotating plasmas [5,6], we have not made any localization approximation here, so that our results are also valid for vortices with sizes comparable to R. Note that when generalized, R represents the local radius of curvature of the vortex trajectory.

Dividing the new polar plane (r', θ') into an inner ($r' < a$) and an outer ($r' > a$) region, we can rewrite (12) and (13) as

$$\Delta\Phi = C_2\left[\Phi - \frac{1}{2}\Omega(r'^2 + 2Rr'\sin\theta' + R^2)\right] - C_1\left[\Psi - \frac{1}{2}\alpha(r'^2 + 2Rr'\sin\theta' + R^2)\right] + C_0 \quad (14)$$

and

$$\Delta\Psi = C_1\left[\Phi - \frac{1}{2}\Omega(r'^2 + 2Rr'\sin\theta' + R^2)\right] + (1 - V_0C_1)\left[\Psi - \frac{1}{2}\alpha(r'^2 + 2Rr'\sin\theta' + R^2)\right]. \quad (15)$$

For simplicity, we shall in the following omit the prime on the new coordinates r' and θ'.

From the localization conditions $\widetilde{\Phi} \to 0$ and $\widetilde{\Psi} \to 0$ as $r \to \infty$, the constants C_0, C_1, and C_2 in the outer region are determined to be

$$C_{0,\mathrm{out}} = 0, \quad (16)$$

$$C_{1,\mathrm{out}} = \frac{-\alpha}{\Omega - \alpha V_0}, \quad (17)$$

and

$$C_{2,\mathrm{out}} = \frac{-\alpha^2}{\Omega(\Omega - \alpha V_0)}. \quad (18)$$

Clearly, the regularity condition of ϕ and A_z in the inner region does not require explicit constraints on C_0, C_1, and C_2. For definiteness, we set $C_{1,\mathrm{in}} = C_{1,\mathrm{out}}$, and leave $C_{0,\mathrm{in}}$ and $C_{2,\mathrm{in}}$ as constant parameters to be determined by the condition requiring the continuity of the perturbed quantities and their derivatives at the boundary of the inner and outer regions.

Substituting (16)–(18) into (14) and (15), and eliminating Ψ, one can obtain the equation governing the perturbed potential Φ in the outer region ($r > a$)

$$\Delta^2\Phi - \frac{1}{\lambda^2}\left(1 - \frac{\alpha^2}{\Omega^2}\right)\Delta\Phi = 0, \quad (19)$$

where $\lambda^2 = (\Omega - \alpha V_0)/\Omega$.

Similarly, we can also get the equation for Φ in the inner region ($r < a$)

$$\Delta^2\Phi - \left[C_{2,\mathrm{in}} + \frac{1}{\lambda^2}\right]\Delta\Phi + \frac{1}{\lambda^2}\left[C_{2,\mathrm{in}} + \frac{1}{\mu^2}\right]\Phi - \frac{1}{2}\Omega a^2\frac{1}{\lambda^2}\left[C_{2,\mathrm{in}} + \frac{1}{\mu^2}\right]\left[\left(\frac{r}{a}\right)^2 + \frac{2Rr}{a}\sin\theta + C'\right] = 0, \quad (20)$$

where $\mu^2 = \lambda^2\Omega^2/\alpha^2$ and

$$C' = \frac{R^2}{a^2} - \frac{4\lambda^2}{a^2} - \frac{2\mu^2 C_{0,\mathrm{in}}}{\Omega a^2(1 + \mu^2 C_{2,\mathrm{in}})}.$$

Assuming that the solutions of (19) and (20) are of the form $\Phi(r,\theta) = \Phi_0(r) + \Phi_1(r)\sin\theta$, we can find the solutions [16]

$$\Phi_{\mathrm{out}}(r,\theta) = \frac{1}{2}\Omega a^2\left[d_1K_0(kr) + \frac{R}{a}\left(d_2K_1(kr) + \frac{d_3}{r}\right)\sin\theta\right], \quad (21)$$

and

$$\Phi_{\mathrm{in}}(r,\theta) = \frac{1}{2}\Omega a^2\left\{g_1J_0(pr) + g_2I_0(qr) + \left(\frac{r}{a}\right)^2 + \left(\frac{b_1}{a}\right)^2 + \frac{R}{a}\left[g_3J_1(pr) + g_4I_1(qr) + \frac{2r}{a}\right]\sin\theta\right\}, \quad (22)$$

where

$$b_1^2 = \frac{4(\lambda^2 C_{2,\mathrm{in}} + 1)}{\mu^2 C_{2,\mathrm{in}}} + C'a^2, \quad (23)$$

$$k^2 = \frac{1}{\lambda^2} - \frac{1}{\mu^2} > 0, \quad (24)$$

$$p^2 = \frac{1}{2}\left\{\left[\left(C_{2,\mathrm{in}} - \frac{1}{\lambda^2}\right)^2 - \frac{4}{\lambda^2\mu^2}\right]^{1/2} - \left(C_{2,\mathrm{in}} - \frac{1}{\lambda^2}\right)\right\}, \quad (25)$$

and

$$q^2 = \frac{1}{2}\left\{\left[\left(C_{2,\mathrm{in}} - \frac{1}{\lambda^2}\right)^2 - \frac{4}{\lambda^2\mu^2}\right]^{1/2} + \left(C_{2,\mathrm{in}} - \frac{1}{\lambda^2}\right)\right\}. \quad (26)$$

Inserting (20) and (21) into (14), we can obtain the expressions for the potential Ψ in the inner and the outer regions

$$\Psi_{\mathrm{out}}(r,\theta) = \frac{1}{2}\alpha a^2\left\{(1 + \mu^2k^2)d_1K_0(kr) + \frac{R}{a}\left[(1 + \mu^2k^2)d_2K_1(kr) + \frac{d_3}{r}\right]\sin\theta\right\} \quad (27)$$

and

$$\Psi_{\rm in}(r,\theta) = \frac{1}{2}\alpha a^2 \Bigg\{ -(C_{2,\rm in}+p^2)\mu^2 g_1 J_0(pr) - (C_{2,\rm in}-q^2)\mu^2 g_2 I_0(qr) + \left(\frac{r}{a}\right)^2 + \left(\frac{b_2}{a}\right)^2 + \frac{R}{a}\left[-(C_{2,\rm in}+p^2)\mu^2 g_3 J_1(pr) - (C_{2,\rm in}-q^2)\mu^2 g_4 I_1(qr) + \frac{2r}{a}\right]\sin\theta\Bigg\}, \tag{28}$$

where d_i $(i=1,2,3)$ and g_j $(j=1,2,3,4)$ are constants to be determined, and

$$b_2^2 = \mu^2\left(4 - C_{2,\rm in}b_1^2\right) + R^2\left(1 + C_{2,\rm in}\mu^2\right) - 2\mu^2 C_{0,\rm in}/\Omega. \tag{29}$$

Recall that r and θ here are the local coordinates with the origin at the center of the vortex. From (20), (21), (27), and (28), it is clear that solutions for the present problem are considerably more complicated than that of the straight-line orbit case. In fact, they consist of both antisymmetric and symmetric parts.

Continuity of Φ, $\partial_r\Phi$, $\Delta\Phi$, Ψ, and $\partial_r\Psi$ at the boundary of the inner and out regions yields the coefficients d_i and g_j, as well as equations determining the inner-region parameters $C_{0,\rm in}$, $C_{2,\rm in}$, and the radius a which represents the characteristic size of the vortex.

The three coefficients associated with the symmetric part of the solution are

$$d_1 = \frac{1}{K_0(ka)\Delta_0}\left[\frac{ka}{K_1(ka)}M\Delta_0 - \frac{p^2+q^2}{k^2-q^2}(EM+GN)\right], \tag{30}$$

$$g_1 = \frac{1}{J_0(pa)\Delta_0}(EM+GN), \tag{31}$$

$$g_2 = \frac{1}{I_0(qa)\Delta_0}(FN-EL), \tag{32}$$

where

$$E = \frac{akK_1(ka)}{K_0(ka)}, \quad F = \frac{apJ_1(pa)}{J_0(pa)}, \quad G = \frac{aqI_1(qa)}{I_0(qa)},$$

$$M = \frac{4 - q^2(a^2+b_1^2)}{a^2(k^2-q^2)}, \quad N = \frac{4 - k^2(a^2+b_1^2)}{a^2(k^2-q^2)},$$

$$L = \frac{4 + p^2(a^2+b_1^2)}{a^2(k^2-q^2)},$$

and

$$\Delta_0 = \frac{p^2+q^2}{k^2-q^2}E + \frac{p^2+k^2}{k^2-q^2}F + G.$$

The four coefficients corresponding to the antisymmetric part of the solution are

$$d_2 = -\frac{4}{K_1(ka)}\frac{p^2+q^2}{k^2\Delta}, \quad d_3 = 2a\left[1 + 2\frac{p^2+q^2}{k^2\Delta}\right], \tag{33}$$

$$g_3 = \frac{4}{J_1(pa)\Delta}, \quad g_4 = -\frac{4}{I_1(qa)\Delta}, \tag{34}$$

where

$$\Delta = \left[\frac{akK_2(ka)}{K_1(ka)} - 2\right]\frac{p^2+q^2}{k^2} + \frac{apJ_2(pa)}{J_1(pa)} + \frac{aqI_2(qa)}{I_1(qa)}.$$

The equations for the parameters $C_{0,\rm in}$, $C_{2,\rm in}$, and the vortex radius a are

$$(1+\mu^2k^2)\frac{ak}{K_1(ka)}M\Delta_0 + \left[(C_{2,\rm in}+p^2)\mu^2 - (1+\mu^2k^2)\frac{p^2+k^2}{k^2-q^2}\right](EM+GN) + (C_{2,\rm in}-q^2)\mu^2(FN-EL) - \left[1 + \left(\frac{b_2}{a}\right)^2\right]\Delta_0 = 0, \tag{35}$$

$$\left[1 + (C_{2,\rm in}+p^2)\mu^2\right]\frac{apJ_2(pa)}{J_1(pa)} + \left[1 + (C_{2,\rm in}-q^2)\mu^2\right]\frac{aqI_2(qa)}{I_1(qa)} - \mu^2(p^2+q^2)\frac{akK_2(ka)}{K_1(ka)} = 0, \tag{36}$$

and

$$[1 + (C_{2,\rm in}+k^2+p^2)\mu^2](EM+GN)F - [1 + (C_{2,\rm in}+k^2-q^2)\mu^2](FN-EL)G - 2\mu^2k^2\Delta_0 = 0. \tag{37}$$

The domain of existence for the angular velocity of the vortex is determined by the relation

$$\frac{1}{\lambda^2} - \frac{1}{\mu^2} > 0, \tag{38}$$

which can be rewritten as

$$\frac{\Omega}{(\Omega - \alpha V_0)} \left(1 - \frac{\alpha^2}{\Omega^2}\right) > 0. \tag{39}$$

Thus, vortices exist if $V_0 < \Omega/\alpha$ and $1 < |\Omega/\alpha|$ as well as when $V_0 > \Omega/\alpha$ and $1 > |\Omega/\alpha|$. The latter condition allows for vortices with $|\Omega/\alpha|$ near zero, a situation impossible in the absence of V_0. Since the lowest-order linear frequency ω for the shear Alfvén waves satisfies $\omega^2 = \alpha^2$, or $\omega = \pm k_\parallel V_A$ in terms of the original (dimensional) parameters, we see that these conditions require $V_0 < \Omega/k_\parallel$ and $|\Omega| > |k_\parallel| V_A$, and $V_0 > \Omega/k_\parallel$ and $|\Omega| < |k_\parallel| V_A$, respectively. That is, the rotation frequency of the vortex must be super-Alfvénic if $V_0 < \Omega/k_\parallel$ and sub-Alfvénic if $V_0 > \Omega/k_\parallel$. It can also be verified that the perturbed magnetic field $\mathbf{B}_\perp = \nabla A_z \times \mathbf{e}_z$ and the parallel current density $j_\parallel = -\Delta A_z/4\pi$ are continuous at $r = a$.

IV. CONCLUSION

In this paper, quasistationary electromagnetic structures resulting from the nonlinear interaction of shear Alfvén waves in low-β ($\beta \ll m_e/m_i$) current-carrying plasmas have been investigated. We derived a system of nonlinear equations describing the interaction of shear Alfvén waves in the cylindrical configuration. Quasistationary localized (in the plane perpendicular to the external magnetic field) asymmetrical vortex-pair solutions with a spatially periodic structure parallel to B_0 are shown to exist. Such vortices can represent a self-organized state of the nonlinear waves. That is, finite amplitude shear Alfvén waves in current-carrying plasmas can self-organize through nonlinear mode-mode coupling to form such vortex structures which can move in sharply curved (circular) orbits. The solution involves symmetric as well as antisymmetric parts [16]. In contrast to the well-known purely antisymmetric local dipole-vortex solution, the vortex here consists of a monopole as well as a dipole, resulting in a vortex pair of unequal intensities. It is also of interest to point out that the inner-region parameters $C_{0,\text{in}}$ and $C_{2,\text{in}}$ as well as the vortex radius a are determined simultaneously by the continuity condition of the perturbed variables (and their derivatives) at $r = a$, so that there are only two free parameters, for example, α and Ω. Finally, we note that when $R \to 0$, the vortex found here degenerates into a *monopole* vortex, while in the limit $R \gg a$ it reduces to a dipole vortex [15].

The present work generalizes the earlier investigations [5,6,15] which involve vortices with straight-line or nearly straight-line orbits. The results here may be applicable in laboratory and space plasmas where self-organized structures with strongly curved trajectories (which, for example, can appear during vortex-vortex interactions) are involved. The collective dynamics and interaction of these vortices may be important in the study of the turbulent magnetized plasma [3,14,17,18]. In fact, many numerical simulations involving vortex generation and interaction in fluids and plasmas seem to indicate the frequent existence of nonsymmetrical vortex pairs [12–14]. The results here can also be useful in interpreting the localized stationary electromagnetic structures in the ionosphere and magnetosphere [11]. On the other hand, for application in higher temperature plasmas, one must include finite gyroradius effects, which can considerably change the properties of the present solutions.

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